

RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. FIRST SEMESTER EXAMINATION, MARCH 2021

FIRST YEAR [BATCH 2020-23]

PHYSICS [HONOURS]

Paper : I [CC1]

Date : 24/03/2021

Time : 11 am - 1 pm

Full Marks : 50

Answer **any five** questions from the following :

[5 × 10]

1. a) Solve the following differential equation

$$y = x + a \tan^{-1} \frac{dy}{dx}$$

- b) Solve the differential equation

$$\frac{d^2y}{dx^2} - 2 \frac{dy}{dx} + y = x e^x \sin x$$

[3+7]

2. a) Solve the following simultaneous equations

$$\frac{dx}{dt} + 2x - 3y = 5t$$

$$\frac{dy}{dt} - 3x + 2y = 2e^{2t}$$

- b) Find out the particular integral by the method of variation of parameters:

$$\frac{d^2y}{dx^2} - 2 \frac{dy}{dx} + y = e^x \log x$$

[7+3]

3. a) Show that the given equation has a regular singularity at $x=0$ and an irregular singularity at $x = \infty$

$$x \frac{d^2y}{dx^2} + (1-x) \frac{dy}{dx} + ay = 0$$

- b) Find the basis of solutions of the following equation by the Frobenius method:

$$x^2 \frac{d^2y}{dx^2} + 4x \frac{dy}{dx} + (x^2 + 2)y = 0$$

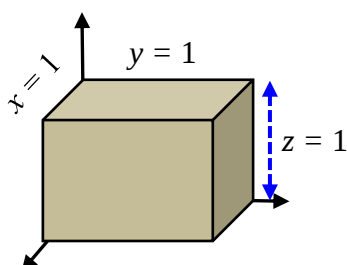
[2+8]

4. a) Find the equation of a plane which is tangent to the surface $x^2 + y^2 - z^2 = 4$ at the point $(1, 2, -1)$.

- b) Find the derivative of the function $\phi = x^2 - 2xy + z^2$ at the point $(2, -1, 1)$ in the direction of the vector $\vec{A} = 2\hat{i} - 4\hat{j} + 4\hat{k}$.

[5+5]

5. a) Given $\vec{A} = 4xz\hat{i} - y^2\hat{j} + yz\hat{k}$, evaluate $\oint \vec{A} \cdot d\vec{\sigma}$ for a unit cube shown below whose surface area is σ .



[1]

- b) Using the definition of Jacobian that you learned in the class, compute an expression for an elemental volume dV in spherical polar coordinates.
- c) Show that the electrostatic force $F(r)$ between two point charges q and q' at a distance r apart is conservative. The electrostatic force is $(r) = \frac{qq'}{r^2} \hat{r}$. [5+2+3]
6. a) Given two set of vectors $S_1 = \{u_1, v_1\}$ and $S_2 = \{u_2, v_2, w_2\}$, where $u_1 = (40, 15)$, $v_1 = (-50, 25)$, $u_2 = (3, 4, 5)$, $v_2 = (2, 9, 2)$ and $w_2 = (4, 18, 4)$. Check whether S_1 and S_2 are linearly independent or dependent.
- b) Given a set of vectors, $S = \{u, v\}$, where $u = (1, 0, 0)$ and $v = (0, 1, 0)$. Does the set of vectors S form a basis of the three dimensional space R^3 . Explain.
- Consider a plane $x + y + z = 5$ in the three dimensional space R^3 . Is this plane a subspace of R^3 . Explain.
- c) Consider a set of vectors $\{(1, 0), (1, -3), (0, 1)\}$ in R^2 . Are they linearly dependent or independent. Explain.
- d) Consider a vector $A = (3, 5)$ in R^2 which is represented in the basis set $\{i, j\}$. [4+2+2+2]
7. a) Find the coordinates of A when represented in another basis set $B = \{u, v\}$ in R^2 , where $u = (1, 1)$ and $v = (0, 2)$.
- b) Consider a set of basis vectors $B = \{(1, 1, 1), (0, 2, 0), (1, 0, 3)\}$. Check whether the basis vectors are orthogonal, if not then derive a set of orthogonal basis vectors from the set B . [5+5]
8. a) What is Cayley Hamilton Theorem on the matrix? Using the Cayley Hamilton Theorem find the inverse of the following matrix

$$\begin{pmatrix} -4 & -2 & 2 \\ 4 & 3 & -4 \\ 0 & -2 & 1 \end{pmatrix}$$

- b) Find Eigenvalues and Eigenvectors of the following matrix

$$\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 1 \end{bmatrix}$$

[3+7]

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